Resonant Lines for Device Measurement, 10 MHz – 3.5 GHz

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BACKGROUND

The resonant line technique began at MIT Electrical Insulation Laboratories during WWII. The method satisfies a basic need to measure low losses in large reactive loads. The technique is to cancel the reactance of the load with a known (calculable) reactance, leaving the losses of the load and the fixture. The fixture losses are assumed to be known, and thus can be separated from load loss.

The supporting mathematics comes directly from transmission line theory, which is well understood. A published standard, ASTM F752–82, was written around the Boonton 34A resonant line. It is not currently in print.

SUMMARY OF THE METHOD

The device under test (DUT) is connected across the conductors of either a short circuited (SC) or open circuited (OC) resonant coaxial transmission line whose electrical length, characteristic impedance, and Q-factor as a function of frequency are assumed to be known. The *in situ* impedance of the DUT causes a change in the resonant frequency and Q-factor of the system. Using transmission line calculations, the resistance and reactance of the DUT are separated from the characteristics belonging to the line.

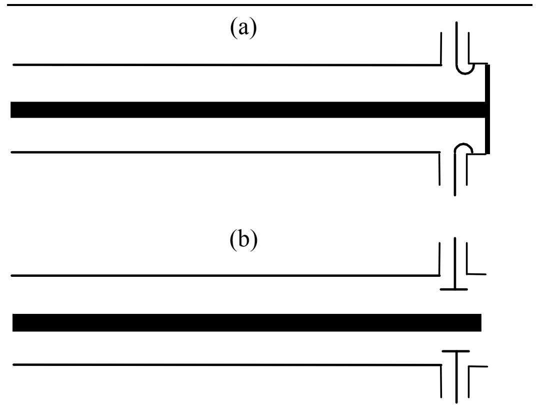

Fig. 1. Loosely coupled transmission lines. (a) Short-circuited termination with inductive coupling. (b) Open circuited termination with capacitive coupling.

As a practical measurement circuit, the transmission line must be connected to a signal source and a detector. This is accomplished by ‘loose coupling’ to the electric field (at the open circuited end) or the magnetic field (at the short-circuited end), fig. 1.

THEORY

Transmission Line Theory

Transmission line theory impedance calculations are based on a physical starting point (the input), a line with known characteristic impedance and electrical length, and a terminating impedance for the line (the load). Thus:

$$Z_i = Z_0 \frac{Z_L + Z_0 \tanh\left[\left(\alpha + j\frac{\omega}{v_{ph}}\right)\ell\right]}{Z_0 + Z_L \tanh\left[\left(\alpha + j\frac{\omega}{v_{ph}}\right)\ell\right]} \qquad \text{(eq. 1)}$$

Here, Z_i is the input impedance being calculated, Z_L is the load impedance, α is the attenuation per unit length, ℓ is the physical length of the line, ω is the angular frequency, v_{ph} is the phase velocity along the line and Z_0 is the line's characteristic impedance.

For a *low loss air dielectric* coaxial transmission line Z_0 can be calculated by the expression:

$$Z_0 \equiv 60 \ln \frac{d_o}{d_i} \qquad \text{(eq. 2)}$$

provided the inside diameter of the outer conductor (d_o) and the outside diameter of the inner conductor (d_i) are known. (Note that this is a low loss approximation.)

Q-factor

The efficiency (quality) of any oscillating system can be expressed as the ratio of energy stored by the system to the power the system dissipates per oscillation (cycle):

$$Q = \frac{\omega\, W_{stored}}{P_{diss}} \qquad \text{(eq. 3)}$$

This ratio is called the quality factor (Q) and can be measured by finding the amplitude resonant frequency (f_r) and dividing by the difference between the two half-power amplitude frequencies (Δf):

$$Q = \frac{f_r}{\Delta f} \qquad \text{(eq. 4)}$$

By energy conservation,

$$P_{system} = P_{DUT} + P_o \qquad \text{(eq. 5)}$$

or equivalently,

$$\frac{\omega W_{system}}{Q_m} = I^2 R_s + \frac{\omega W_0}{Q_0} \qquad \text{(eq. 6)}$$

Q_m is the measured Q-factor of the system with the DUT in place, Q_0 is the quality factor of the empty transmission line and I^2R_s is the power dissipated in the DUT.

By the energy stored, power dissipated definition, a capacitor's Q-factor may be expressed as:

$$Q = \frac{\omega C}{G} \qquad \text{(eq. 7)}$$

or its Thevenin equivalent:

$$Q = \frac{1}{\omega C R_s} \qquad \text{(eq. 8)}$$

For a low loss transmission line, where L_0 and R_0 are the inductance and resistance per unit length, and ω_0 is the fundamental resonant angular frequency, eq. 3 can be restated:

$$Q = \frac{\omega L_0}{R_0} \qquad \text{(eq. 9)}$$

Q-factor of a Physical Transmission Line

While it may be assumed that L_0 in eq. 9 changes very little with frequency, a change in R_0 due to the phenomenon of skin depth must be counted. Hence:

$$Q_0(\omega) = Q_0(\omega_0)\sqrt{\omega/\omega_0} \qquad \text{(eq. 10)}$$

Here, $Q_0(\omega_0)$ is the Q-factor of the line at the fundamental resonant frequency, while $Q_0(\omega)$ is the line at a different frequency.

Q-factor of a Coupled Transmission Line

The resonant line is made useful as a measurement tool by loosely coupling it to a signal source and a detector, fig. 1. Through the coupling circuit, energy is imparted to the resonant system, but energy can be imparted from the resonator to the external circuitry as well. Thus, the Q-factor of the external system must be counted along with the Q-factor of the transmission line.

Because power obeys the superposition principle, eq. 3 may be modified, as follows:

$$Q_{total} = \frac{\omega W_{stored}}{P_0 + P_{ext}} \qquad \text{(eq. 11)}$$

Assume that the external circuit does not store energy, so that W_{stored} is entirely due to the resonant line. In this case, eq. 11 may be inverted and separated, thus:

$$\frac{1}{Q_{total}} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \qquad \text{(eq. 12)}$$

Define the *coupling coefficient* (κ) as the ratio of Q_0 to Q_{ext}, substitute eq. 10 for Q_0 and rearrange eq. 11, so that:

$$Q_L(\omega) = Q_0(\omega_0)\sqrt{\omega/\omega_0}\,\frac{1}{1+\kappa} \qquad \text{(eq. 13)}$$

The coupling coefficient is itself a function of frequency, making eq. 13 impractical. An alternative is to measure $Q_L(\omega)$ at several frequencies and choose a fitting function based on a series expansion of eq. 13. The fitting function given by ASTM F752–82 is:

$$Q_L = Q_0(\omega/\omega_0)^x \qquad \text{(eq. 14)}$$

where x is defined as:

$$x \equiv \frac{\log(Q_1/Q_0)}{\log(f_1/f_0)},$$

and f_1 and Q_1 are the second harmonic and the measured Q at the second harmonic, respectively.

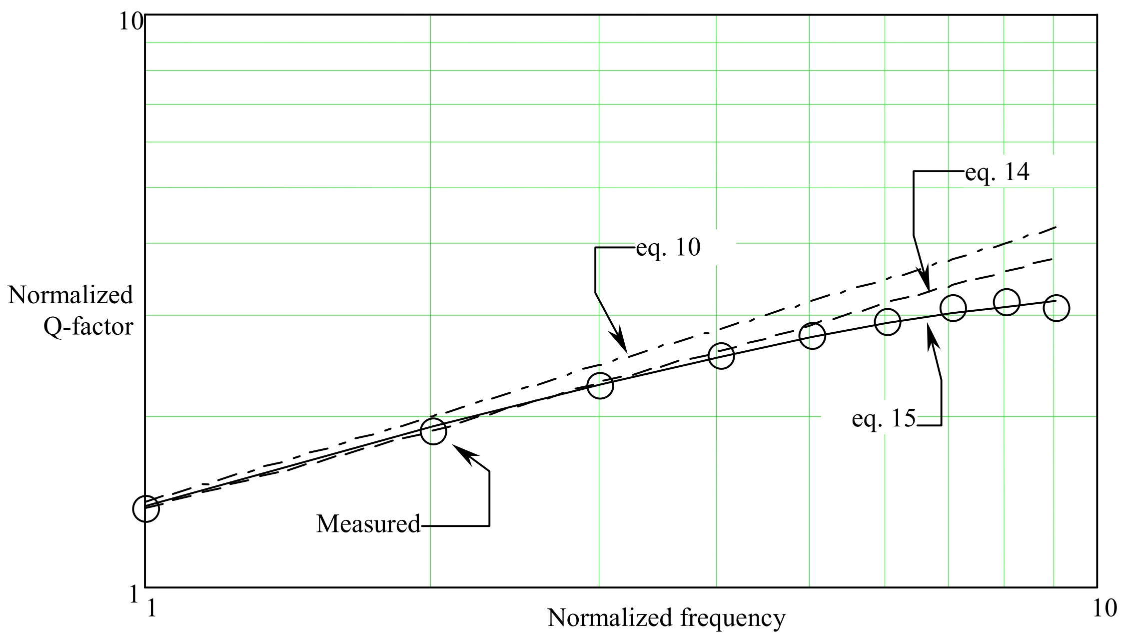

Fig. 2. Normalized Q-factor vs. normalized frequency of Boonton 34A resonant line, S/N 176. Circles represent measured data points. Lines represent interpolation equations, forced through $Q(f_0)$.

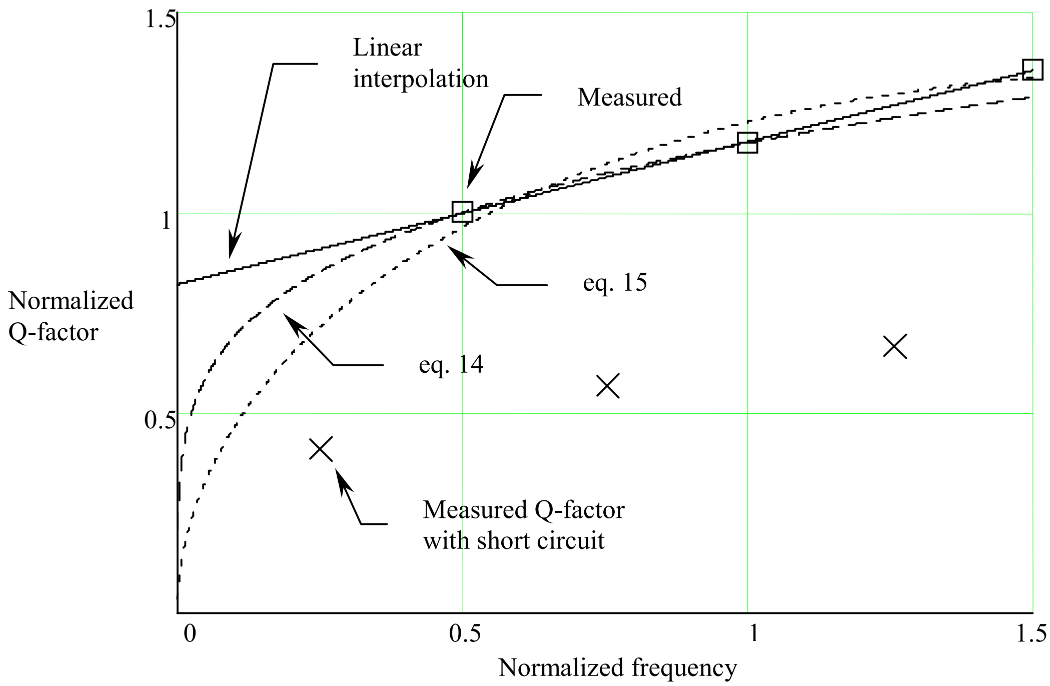

Fig. 3. Normalized Q-factor vs. normalized frequency of Ingalls Engineering 15 cm resonant line, S/N 15c-002. Squares represent measured Q factor of empty line. Solid line is a linear interpolation of measured data, while the dotted and dashed lines are power series interpolations (eqs. 14 and 15). Crosses represent Q-factor of line with a short circuit as a DUT.

When measured Q_L has more curvature over the frequencies of interest, a two term power series may be preferable:

$$Q_L = Q_0\left[\gamma_1(\omega/\omega_0)^{1/2} - \gamma_2(\omega/\omega_0)^{3/2}\right] \qquad \text{(eq. 15)}$$

The quantities γ_1 and γ_2 are chosen to fit the experimental data.

When measured Q_L has less curvature with respect to frequency, a linear interpolation may be a better choice than either eqs. 14 or 15. Again, the slope and intercept are chosen to best fit the data, fig. 3.

Extraction of R_s

The extraction of R_s begins with the rearrangement of eq. 6:

$$R_s = \frac{1}{|I(z)|^2}\left(\frac{\omega W_{sys}}{Q_m} - \frac{\omega W_0}{Q_0}\right) \qquad \text{(eq. 16)}$$

The energy stored in a transmission line is:

$$W_{line} = \tfrac{1}{4}\int_0^{\ell} L_0 \left|I(z)\right|^2 \partial z + \tfrac{1}{4}\int_0^{\ell} C_0 \left|V(z)\right|^2 \partial z \qquad \text{(eq. 17)}$$

Here, the DUT's location is defined to be at z = 0(at the *input* of the line) and the length of the transmission line is ℓ. The *load impedance*, either an open or short circuit, is located at $z = \ell$.

The energy stored in the line / DUT combination is:

$$W_{sys} = \tfrac{1}{4}\int_0^{\ell} L_0 |I(z)|^2 \partial z + L_{DUT}|I(0)|^2 + \tfrac{1}{4}\int_0^{\ell} C_0 |V(z)|^2 \partial z + C_{DUT}|V(0)|^2 \qquad \text{(eq. 18)}$$

Since this is a resonant system, the energy stored in the magnetic field must necessarily equal the energy stored in the electric field. Thus,

$$W_{sys} = \tfrac{1}{2}\int_0^{\ell} L_0 |I(z)|^2 \partial z + L_{DUT}|I(0)|^2$$
or
$$W_{sys} = \tfrac{1}{2}\int_0^{\ell} C_0 |V(z)|^2 \partial z + C_{DUT}|V(0)|^2 \qquad \text{(eq. 19)}$$

Extraction of Reactance

Extraction of reactance is much easier in principle because only eq. 1 is involved. At resonance, the reactance of the transmission line is equal in magnitude and opposite in sign to the reactance of the DUT. Again, the calculation simplifies due to the boundary conditions. In practice, separation of the DUT reactance from the discontinuity reactance of the DUT / line connection is non-trivial, making the reactance measurement less accurate than the loss measurement. Fortunately, the discontinuity reactance is 'lossless' so that the measurement of R_s is unaffected.

Calculations for the Short Circuited Configuration

In the short-circuited (SC) configuration, the DUT is connected across the conductors of a transmission line that is terminated in a short circuit. Thus, the load impedance, Z_L, is zero. At resonance, the input reactance of the line and the reactance of the DUT must be equal in magnitude and opposite in sign. The reactance of the DUT, from eq. 1 simplifies to:

$$X_{DUT} = -Z_0 \tan\left(\frac{\pi}{2}\frac{f}{f_0}\right) \qquad \text{(eq. 20)}$$

Here, $\omega_0\ell/v_{ph}$ has been normalized to $(\pi/2)$, where ω_0 is the quarter wave resonance (angular) frequency of the empty line.

The loss of the DUT is derived from eqs. 16 – 19. The result is:

$$R_{DUT} = Z_0\left[\frac{\pi}{4}\frac{f}{f_0}\sec^2\left(\frac{\pi}{2}\frac{f}{f_0}\right) + \frac{1}{2}\tan\left(\frac{\pi}{2}\frac{f}{f_0}\right)\right]\left(\frac{1}{Q_m} - \frac{1}{Q_L}\right) \qquad \text{(eq. 21)}$$

From this quantity, a "fixture resistance," essentially the contact resistance between line and DUT, is usually subtracted, see below.

Calculations for the Open Circuited Configuration

In the open circuited (OC) configuration, the DUT is connected across the conductors of a transmission line that is terminated in an open circuit. Thus, the load impedance, Z_L, is infinity. The open and short-circuited resonant lines are dual cases of one another. Thus, we may write:

$$X_{DUT} = Z_0 \cot\left(\frac{\pi}{2}\frac{f}{f_0}\right) \qquad \text{(eq. 22)}$$

and

$$R_{DUT} = Z_0\left[\frac{\pi}{4}\frac{f}{f_0}\csc^2\left(\frac{\pi}{2}\frac{f}{f_0}\right) - \frac{1}{2}\cot\left(\frac{\pi}{2}\frac{f}{f_0}\right)\right]\left(\frac{1}{Q_m} - \frac{1}{Q_L}\right) \qquad \text{(eq. 23)}$$

Again, fixture resistance is usually subtracted from R_{DUT}.

Fixture Resistance

When the DUT is a short circuit, the measured Q-factor of the line is lower than predicted by any of the interpolation rules discussed above, fig. 3. This difference is usually assigned to contact resistance, although there is some evidence that the coupling coefficient also changes. If contact resistance was the major contributor to the lowered Q-factor, then skin depth behavior of fixture resistance would a reasonable expectation. Thus, we could write:

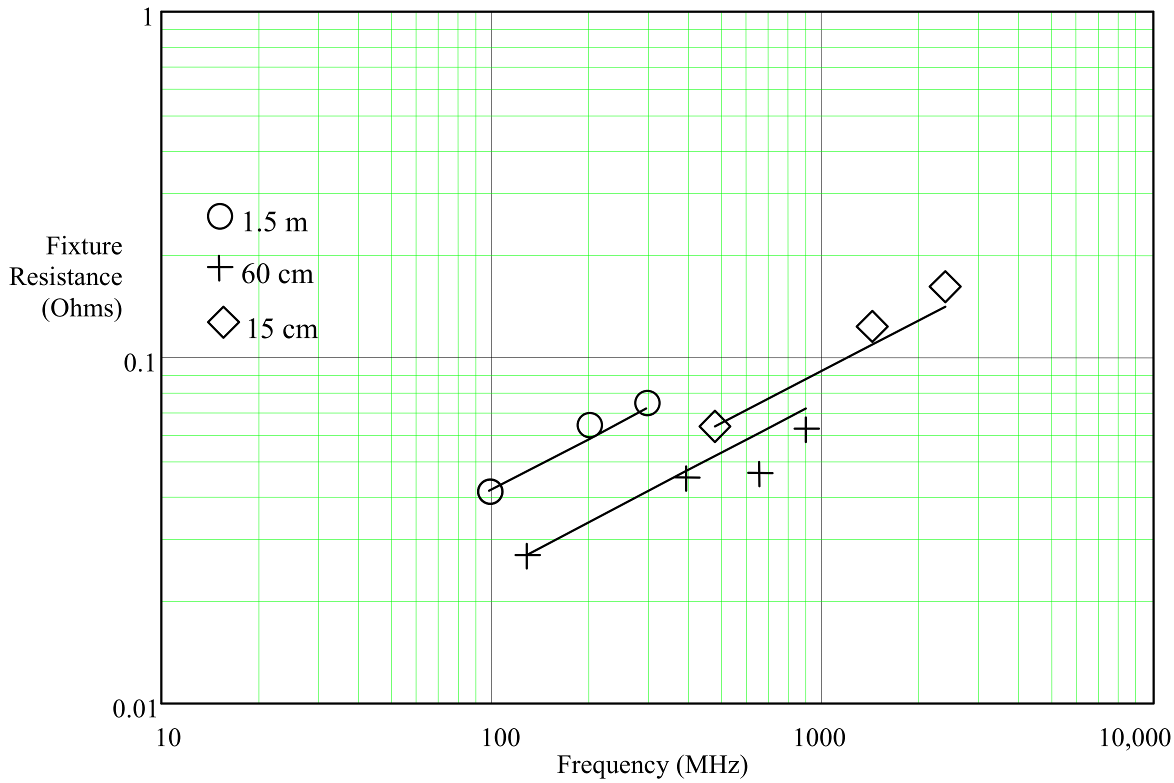

Fig. 4. Calculated fixture resistance for three resonant lines with skin depth rule applied to each result.

$$R_{fix}(f) = R_{fix}(f_0)\sqrt{f/f_0} \qquad \text{(eq. 24)}$$

The results depicted in fig. 4 lend credence to the supposition that fixture resistance can indeed be modeled as a skin depth phenomenon. This is a departure from ASTM F752–82 , which uses the equation:

$$R_{fix}(f) = R_{fix}(f_0)(f/f_0)^{1-x}$$

without justification.

DESIGN AND CONSTRUCTION

A 1.5 m RF Line (~10 to ~300 MHz)

One challenge presented by lower frequency resonant line measurements is finding space for the test fixture, fig. 5. An OC line configuration makes the problem worse (for capacitors) because the test frequency will be higher for a given length of line. Therefore, the SC configuration is the obvious choice for RF testing.

The 1.5 m line's inner and outer conductors are hard drawn copper pipes. The center conductor, nominally 1 ¼ in., had an outside diameter of 3.035 cm. The outer conductor, nominally 4 in., had an inside diameter of 10.21 cm. The resulting Z_0 was 72.75 Ω. The line is shown in fig. 5.

The theoretical Q-factor, calcu-

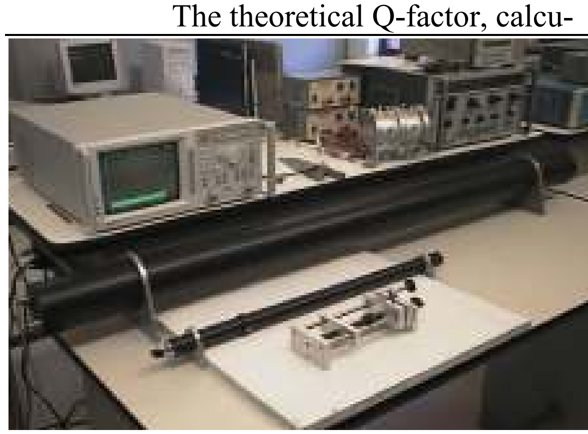

Fig. 5. Three resonant lines: 1.5 m (background), 60 cm (center) and 15 cm (foreground).

lated from dimensional quantities and the conductivity of copper, was about twice the measured Q-factor of the line at f_0.

The preferred coupling method is to use loops at the short-circuited end. The loops used for the 1.5 m line are ~7.5 cm long and ~1.5 cm wide. They were fabricated by soldering pre-shaped loops of (un-insulated) 14 ga. Cu wire to the center conductor and shield of a pair of N-type bulkhead receptacles.

Over-coupling can decrease the loaded Q-factor of the line and also perturb the resonance frequencies. Under-coupling makes it difficult to measure large capacitance values due to weak signal strength. A good compromise is to adjust the coupling by rotating the loops until the insertion loss at f_0 is ~30 dB.

The DUT is connected to the line by means of a wedge fixture, fig. 6. The wedge fixture has four separate copper parts, all of which are rhodium plated for wear resistance. Two of the parts are soldered to inner and outer conductors. The other pair, with a larger gap between them is attached with screws to the first. Small DUTs are wedged between the first pair, while larger DUTs are wedged between the second pair. Leaded DUTs are clamped between the inner and outer pairs.

Radiation losses at the DUT end of the line are reduced by adding a section of circular waveguide. Because the line is operated below the cutoff frequency of the waveguide, the guide is effectively an open circuit in parallel with the DUT.

Fig. 6. Wedge fixture of 1.5m SC line. Note circular waveguide section, hinged to the support, at top.

A 60 cm UHF Line (~125 to ~900 MHz)

Fig. 7 depicts a 60 cm OC line. It is similar to the Boonton 34A in its frequency range and in the size of the inner and outer conductors. It differs from Boonton's design in coupling and in the way a DUT is connected.

The line is capacitively coupled at its open-circuited end by two modified SMA sparkplug launchers. The sparkplugs were modified by soldering ~5 cm diameter brass disks on the launcher tips to increase the coupling to the center conductor.

The center conductor is fabricated from nominal $^3/_8$ in. OFHC copper rod, while the outer conductor is nominal 1 ¼ in. hard drawn copper tubing. The center conductor is tapped on the open-circuited end to accept a $^3/_8$ in. Teflon rod with a ¼-20 threaded nylon insert. A lead-screw, compression spring and thrust bushing combination drives the Teflon rod, enabling the center conductor to move back and forth inside the line. This allows the DUT to be clamped between the center conductor and a shorting plate at the DUT end of the line. The shorting plate and the end of the center conductor are rhodium plated for wear resistance.

The DUT is inserted through a port milled into the line and the lead-screw is turned until the center conductor clamps the DUT to the shorting plate.

The measured Q factor at 2 f_0 is about 30% of theoretical.

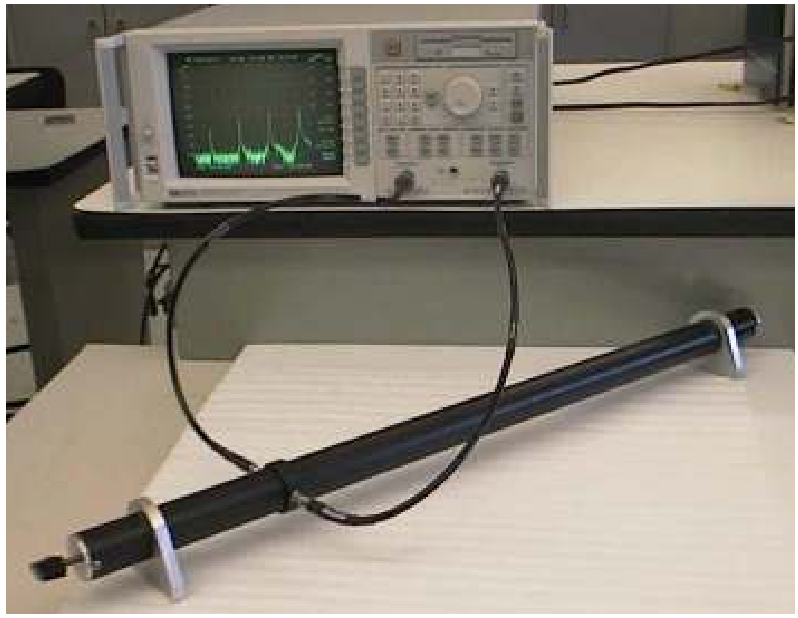

Fig. 7. 60 cm OC resonant line, connected to HP 8714C RF VNA. The lead-screw, left foreground actuates a dielectric rod, which drives the center conductor. The DUT is inserted through a port in the opposite end. Note the resonances displayed on the VNA

A 15 cm L/S-band Line (~0.5 to ~3.5 GHz)

A 15 cm resonant line was designed for compatibility with EIA size 0805 and smaller DUTs. The goal was to reduce discontinuity reactance at the DUT connection point, allowing improved measurement accuracy. Now, the spatial problem changes from fitting the line into the lab, to fitting a DUT into the line.

Fig. 8 shows the line, cradled in a twin lead-screw assembly. The line itself is only slightly larger in diameter than the armored microwave cables connecting it to the VNA. The outer conductor is pressed into two aluminum carrier plates that ride on a pair of stainless steel guide rails. These rails are fitted to end plates that are normal to the axis of the line. At the DUT end of the line, the end plate accepts a soft conducting shim. This shim, gold plated copper foil on soft circuit board, allows for minor irregularities in the machined end of the outer conductor, which must be slid away from the shim to allow the DUT to be inserted into the line. The shim is replaceable.

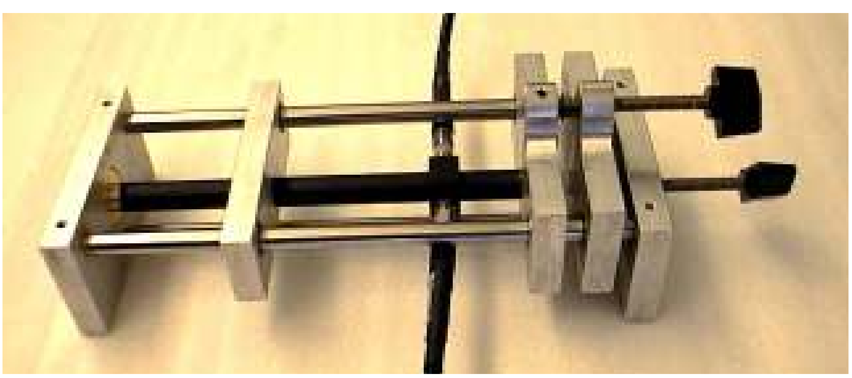

Fig. 8. 15 cm OC resonant line. SMA terminated cables, in foreground and background, connect the line to the VNA. The device is inserted at the left. The top lead-screw, at right, moves the outer conductor, while the bottom lead-screw moves both conductors.

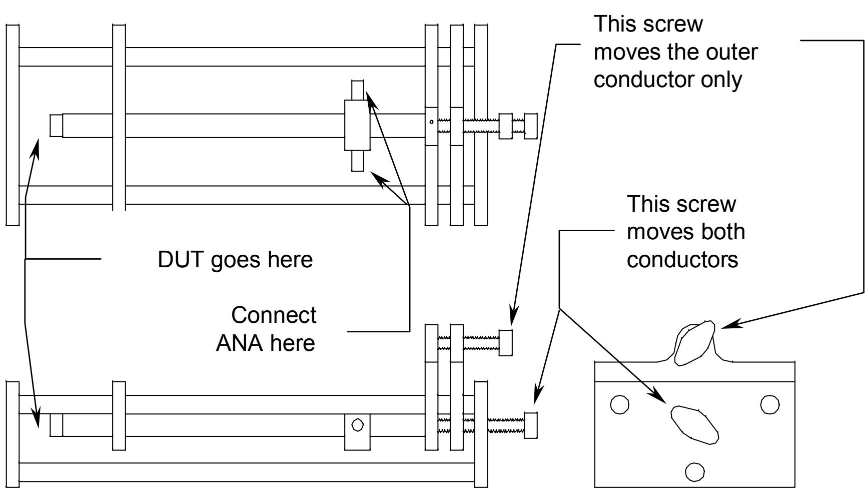

Fig. 9. Plan view of 15 cm resonant line. The DUT is placed onto the center conductor and the lead-screw that moves both conductors is turned until the DUT is clamped between the center conductor and shim. The outer conductor is then brought into contact with the shim by turning the lead-screw that moves the outer conductor only.

The center conductor was rhodium plated on both ends for wear resistance. It is intended that when the plating on the DUT end of the center conductor wears out, it can be removed from the line and flipped, providing a fresh contact surface to the DUT.

The outer conductor is an 11 mm diameter electro-formed OFHC copper tube which is rhodium plated at the DUT end.

Z_0 is 75.03 Ω the measured Q at 2 f_0 is about 40% of theoretical.

On the opposite end from the DUT are two lead-screws. One lead-screw is threaded through the end plate and is captured in a sliding thrust plate that moves both the inner and outer conductor. A second lead-screw is threaded though the thrust plate and is captured in a sliding carrier plate which in turn is pressed onto the outer conductor. This second lead-screw closes the outer conductor over the DUT after it has been clamped between the inner conductor and the soft shim, fig. 9.

Fig. 10 shows a DUT that has just been clamped between the center conductor and the shim. Next, the outer conductor would be moved over the DUT until it also contacted the shim. At that point, the DUT would be ready to measure.

The center conductor is 3.18 mm diameter OFHC copper rod, which has been stepped down on both ends to a diameter of 1.83 mm to allow for the presence of solid Teflon bushings at the ends while maintaining a constant impedance.

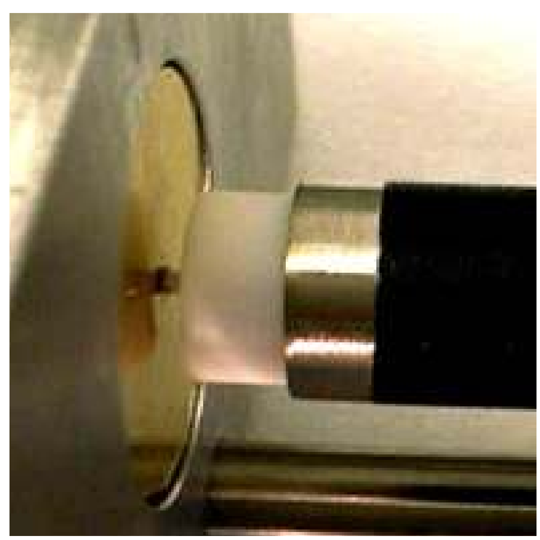

Fig. 10. EIA size 0504 device captured between center conductor and shim of 15 cm line. Note rhodium plating on outer conductor and solid Teflon bushing.

DEVICE MEASUREMENTS

Hundreds of capacitors from seven manufacturers have been measured to date using these lines. An 18 pF EIA size 0603 capacitor, measured with the 1.5 m, 60 cm, and 15 cm resonant lines, is here presented as a typical result. Series resistance is presented in fig. 11 and equivalent capacitance is presented in fig. 12. Evidently, the device is (series) self-resonant at ~1 GHz.

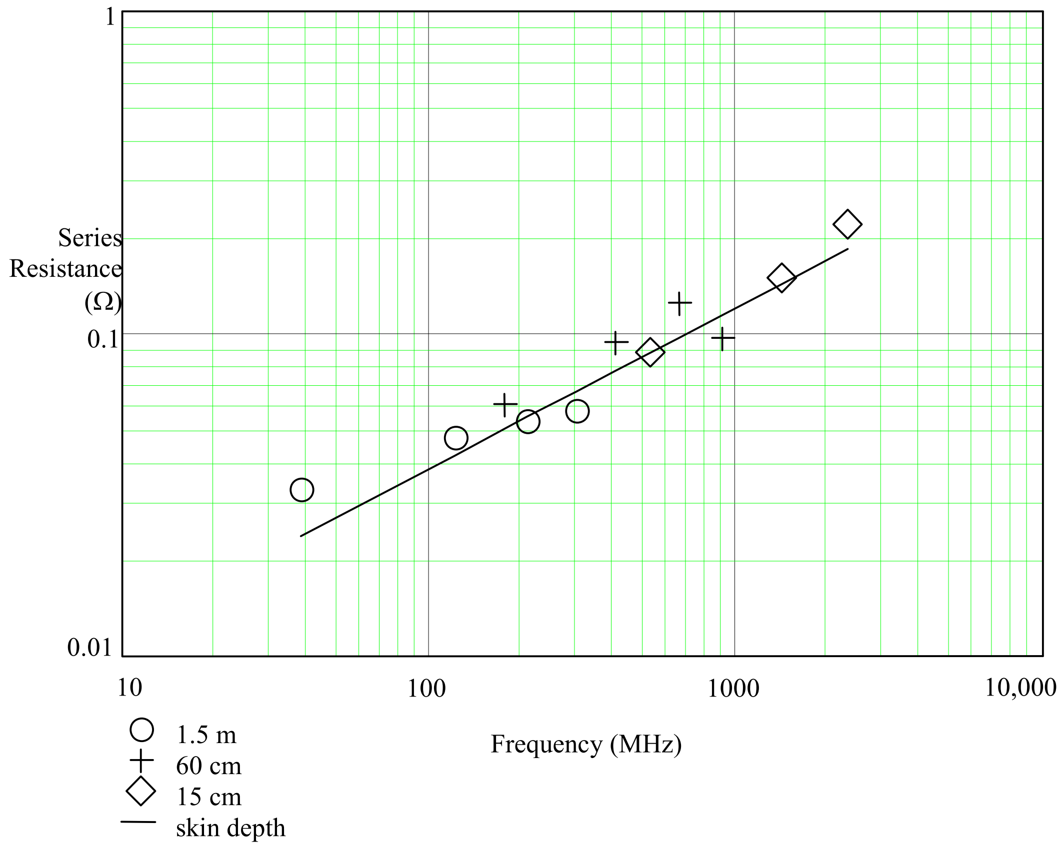

Fig. 12. Series resistance of 18 pF MLC, measured on three different lines.

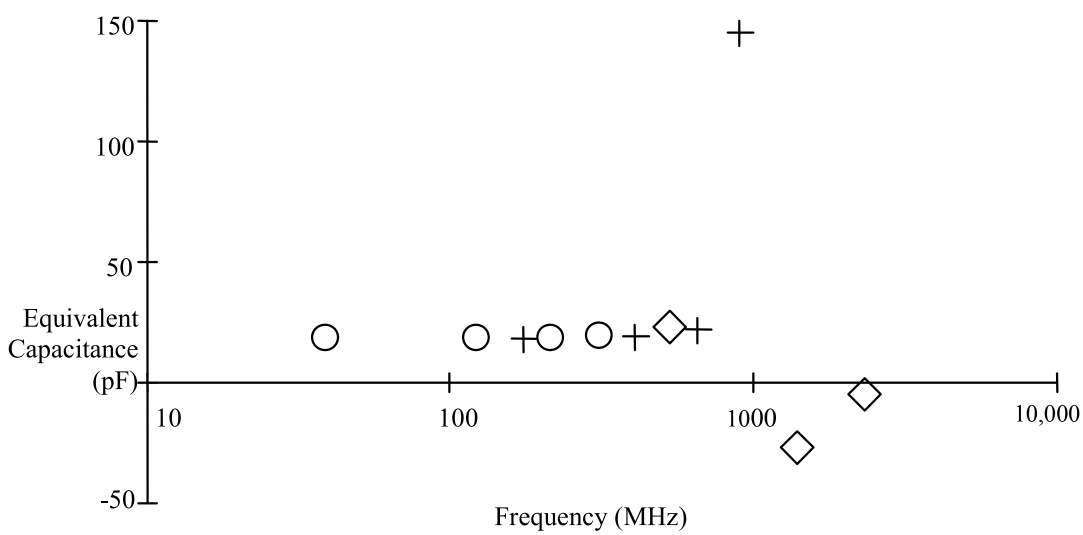

Fig. 11. Equivalent capacitance of 18 pF MLC, measured on three different lines.

The technique is not limited to capacitors. Four EIA size 0805 wire wound inductors were tested with the 15 cm line. A SPICE equivalent circuit was written to model all of the inductors using the 15 cm line data, fig. 13. The element values of Rp = 50 kΩ, Cp = 135 fF, and Rs = 1 Ω (skin depth frequency of 1 GHz) were found to work satisfactorily for all four DUTs, fig. 14.

CONCLUSION

The resonant line technique, one of the oldest methods of high frequency device characterization, is still an important tool for evaluating reactive components. Modern network analyzers and computers have not made the technique obsolete, but have instead helped to make the test procedure faster, easier and more accurate.

Three resonant lines were designed and constructed by the author.

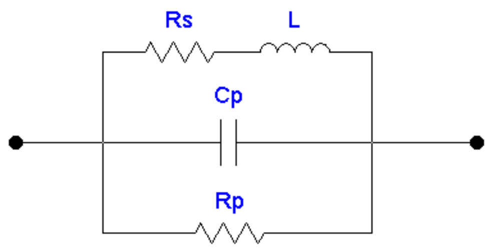

Fig. 13. SPICE equivalent circuit for four EIA size 0805 wire-wound inductors. Rs (1 Ω), Cp (135 fF), and Rp (50 kΩ), are the same for all inductors tested, based upon best fit to all measured data.

Improvements to interpolation of Q-factor, frequency range, and DUT connection have been demonstrated in the measured results of both capacitors and coils.

The technique is a useful companion to other high frequency measurements, especially for low loss capacitors. The method deserves an updated standard.

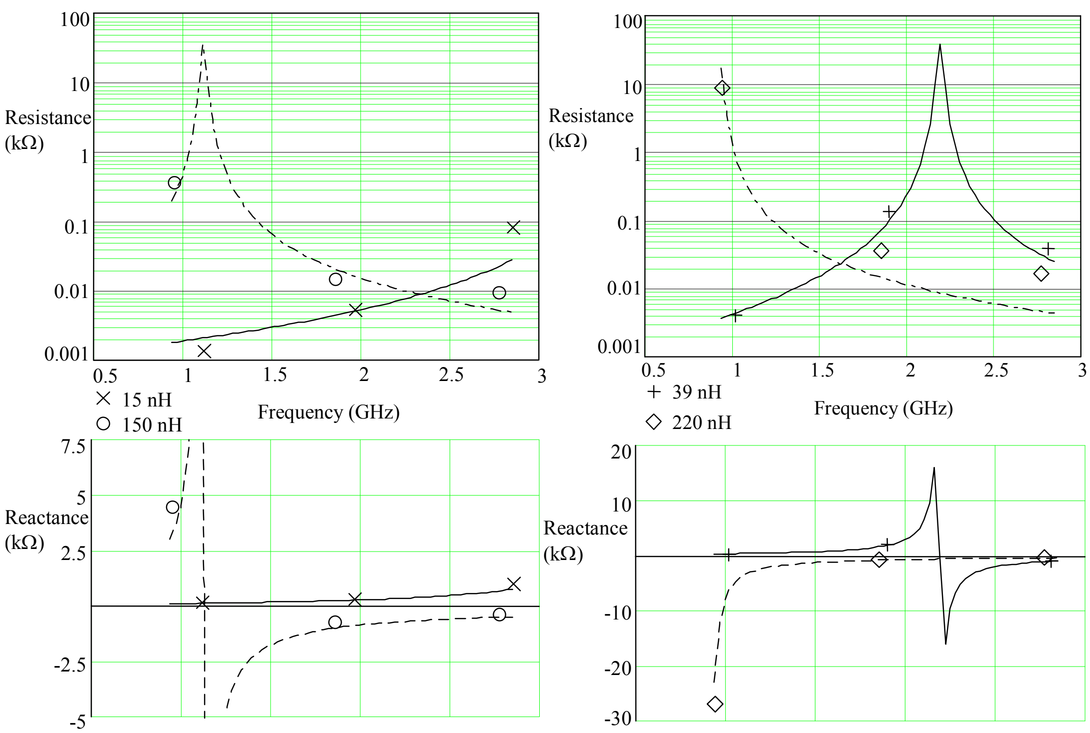

Fig. 14. Resistance and reactance of 15, 150, 39, and 220 nH, wire-wound inductors. Curves are based on measured inductance values and best fit to SPICE parameters for all DUTs, fig. 13.

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